DPP - Daily Practice Problems

Chapter-wise Sheets

Date : ______ **Start Time :** ______ **End Time :** ______

PHYSICS

CP06

SYLLABUS : System of Particles and Rotational Motion

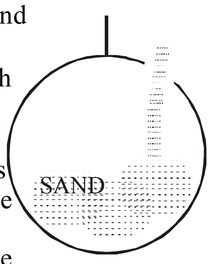
Max. Marks : 180 **Marking Scheme :** (+4) for correct & (–1) for incorrect answer **Time : 60 min.**

INSTRUCTIONS : This Daily Practice Problem Sheet contains 45 MCQs. For each question only one option is correct. Darken the correct circle/ bubble in the Response Grid provided on each page.

1. From a solid sphere of mass M and radius R, a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is :

(a) $\frac{4MR^2}{9\sqrt{3}\pi}$ (b) $\frac{4MR^2}{3\sqrt{3}\pi}$ (c) $\frac{MR^2}{32\sqrt{2}\pi}$ (d) $\frac{MR^2}{16\sqrt{2}\pi}$

2. A hollow sphere is held suspended. Sand is now poured into it in stages.
The centre of mass of the sphere with the sand



(a) rises continuously
(b) remains unchanged in the process
(c) first rises and then falls to the original position
(d) first falls and then rises to the original position

3. A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass $\frac{1}{3}M$ and a body C of mass $\frac{2}{3}M$. The centre of mass of bodies B and C taken together shifts compared to that of body A towards

(a) does not shift
(b) depends on height of breaking
(c) body B (d) body C

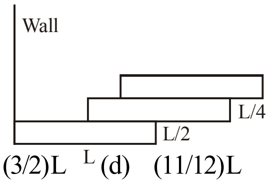
4. From a uniform wire, two circular loops are made (i) P of radius r and (ii) Q of radius nr. If the moment of inertia of Q about an axis passing through its centre and perpendicular to its plane is 8 times that of P about a similar axis, the value of n is (diameter of the wire is very much smaller than r or nr)

(a) 8 (b) 6 (c) 4 (d) 2

5. A billiard ball of mass m and radius r, when hit in a horizontal direction by a cue at a height h above its centre, acquired a linear velocity v_0. The angular velocity ω_0 acquired by the ball is

(a) $\frac{5v_0r^2}{2h}$ (b) $\frac{2v_0r^2}{5h}$ (c) $\frac{2v_0h}{5r^2}$ (d) $\frac{5v_0h}{2r^2}$

6. Three bricks each of length L and mass M are arranged as shown from the wall. The distance of the centre of mass of the system from the wall is



(a) L/4 (b) L/2 (c) (3/2)L (d) (11/12)L

7. Four point masses, each of value m, are placed at the corners of a square $ABCD$ of side ℓ. The moment of inertia of this system about an axis passing through A and parallel to BD is

(a) $2m\ell^2$ (b) $\sqrt{3}m\ell^2$ (c) $3m\ell^2$ (d) $m\ell^2$

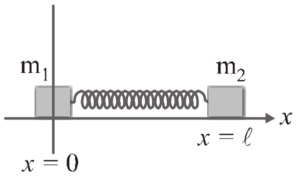
8. A loop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip?

(a) $\frac{r\omega_0}{4}$ (b) $\frac{r\omega_0}{3}$ (c) $\frac{r\omega_0}{2}$ (d) $r\omega_0$

RESPONSE GRID					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ		

Space for Rough Work

9. Two masses m_1 and m_2 are connected by a massless spring of spring constant k and unstretched length ℓ. The masses are placed on a frictionless straight channel, which are consider our x-axis. They are initially at $x = 0$ and $x = \ell$ respectively. At $t = 0$, a velocity v_0 is suddenly imparted to the first particle. At a later time t, the centre of mass of the two masses is at :



(a) $x = \frac{m_2 \ell}{m_1 + m_2}$

(b) $x = \frac{m_1 \ell}{m_1 + m_2} + \frac{m_2 v_0 t}{m_1 + m_2}$

(c) $x = \frac{m_2 \ell}{m_1 + m_1} + \frac{m_2 v_0 t}{m_1 + m_2}$ (d) $x = \frac{m_2 \ell}{m_1 + m_2} + \frac{m_1 v_0 t}{m_1 + m_2}$

10. A body of mass 1.5 kg rotating about an axis with angular velocity of 0.3 rad s^{-1} has the angular momentum of 1.8 kg $m^2 s^{-1}$. The radius of gyration of the body about an axis is
(a) 2m (b) 1.2m (c) 0.2m (d) 1.6m

11. If $\vec{F}$ is the force acting on a particle having position vector $\vec{r}$ and $\vec{\tau}$ be the torque of this force about the origin, then:

(a) $\vec{r} \cdot \vec{\tau} > 0$ and $\vec{F} \cdot \vec{\tau} < 0$

(b) $\vec{r} \cdot \vec{\tau} = 0$ and $\vec{F} \cdot \vec{\tau} = 0$

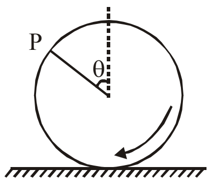
(c) $\vec{r} \cdot \vec{\tau} = 0$ and $\vec{F} \cdot \vec{\tau} \neq 0$

(d) $\vec{r} \cdot \vec{\tau} \neq 0$ and $\vec{F} \cdot \vec{\tau} = 0$

12. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of

(a) $\frac{1}{6}\frac{l\omega}{g}$ (b) $\frac{1}{2}\frac{l^2\omega^2}{g}$ (c) $\frac{1}{6}\frac{l^2\omega^2}{g}$ (d) $\frac{1}{3}\frac{l^2\omega^2}{g}$

13. A wheel is rolling straight on ground without slipping. If the axis of the wheel has speed v, the instantenous velocity of a point P on the rim, defined by angle θ, relative to the ground will be



(a) $v\cos\left(\frac{1}{2}\theta\right)$ (b) $2v\cos\left(\frac{1}{2}\theta\right)$

(c) $v(1+\sin\theta)$ (d) $v(1+\cos\theta)$

14. A solid sphere having mass m and radius r rolls down an inclined plane. Then its kinetic energy is

(a) $\frac{5}{7}$ rotational and $\frac{2}{7}$ translational

(b) $\frac{2}{7}$ rotational and $\frac{5}{7}$ translational

(c) $\frac{2}{5}$ rotational and $\frac{3}{5}$ translational

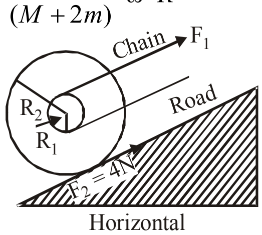
(d) $\frac{1}{2}$ rotational and $\frac{1}{2}$ translational

15. A ring of mass M and radius R is rotating about its axis with angular velocity ω. Two identical bodies each of mass m are now gently attached at the two ends of a diameter of the ring. Because of this, the kinetic energy loss will be :

(a) $\frac{m(M+2m)}{M}\omega^2 R^2$ (b) $\frac{Mm}{(M+m)}\omega^2 R^2$

(c) $\frac{Mm}{(M+2m)}\omega^2 R^2$ (d) $\frac{(M+m)M}{(M+2m)}\omega^2 R^2$

16. Acertain bicycle can go up a gentle incline with constant speed when the frictional force of ground pushing the rear wheel is $F_2 = 4$ N. With what force F_1 must the chain pull on the sprocket wheel if $R_1 = 5$ cm and $R_2 = 30$ cm?

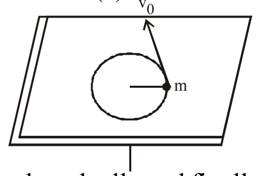


(a) 4 N (b) 24 N (c) 140 N (d) $\frac{35}{4}$ N

17. A wooden cube is placed on a rough horizontal table, a force is applied to the cube. Gradually the force is increased. Whether the cube slides before toppling or topples before sliding is independent of :
(a) the position of point of application of the force
(b) the length of the edge of the cube
(c) mass of the cube
(d) Coefficient of friction between the cube and the table

18. From a circular ring of mass M and radius R, an arc corresponding to a 90° sector is removed. The moment of inertia of the ramaining part of the ring about an axis passing through the centre of the ring and perpendicular to the plane of the ring is k times MR^2. Then the value of k is
(a) 3/4 (b) 7/8 (c) 1/4 (d) 1

19. A mass m moves in a circle on a smooth horizontal plane with velocity v_0 at a radius R_0. The mass is attached to string which passes through a smooth hole in the plane as shown.



The tension in the string is increased gradually and finally m moves in a circle of radius $\frac{R_0}{2}$. The final value of the kinetic energy is

(a) $\frac{1}{4}mv_0^2$ (b) $2mv_0^2$ (c) $\frac{1}{2}mv_0^2$ (d) mv_0^2

20. A rod PQ of length L revolves in a horizontal plane about the axis YY′. The angular velocity of the rod is ω. If A is the area of cross-section of the rod and ρ be its density, its rotational kinetic energy is

RESPONSE GRID				
9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ
14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ
19. ⓐⓑⓒⓓ				

Space for Rough Work

(a) $\frac{1}{3}AL^3\rho\omega^2$ (b) $\frac{1}{2}AL^3\rho\omega^2$

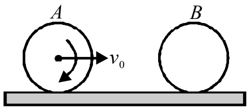
(c) $\frac{1}{24}AL^3\rho\omega^2$ (d) $\frac{1}{18}AL^3\rho\omega^2$

21. A solid sphere of mass 2 kg rolls on a smooth horizontal surface at 10 m/s. It then rolls up a smooth inclined plane of inclination 30° with the horizontal. The height attained by the sphere before it stops is

(a) 700 cm (b) 701 cm (c) 7.1 m (d) 70 m

22. A hollow smooth uniform sphere *A* of mass m rolls without sliding on a smooth horizontal surface. It collides head on elastically with another stationary smooth solid sphere *B* of the same mass m and same radius. The ratio of kinetic energy of *B* to that of *A* just after the collision is

(a) 1 : 1
(b) 2 : 3
(c) 3 : 2
(d) 4 : 3



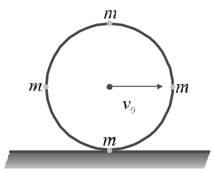
23. Two discs of same thickness but of different radii are made of two different materials such that their masses are same. The densities of the materials are in the ratio of 1 : 3. The moments of inertia of these discs about the respective axes passing through their centres and perpendicular to their planes will be in the ratio of

(a) 1 : 3 (b) 3 : 1 (c) 1 : 9 (d) 9 : 1

24. A pulley fixed to the ceiling carries a string with blocks of mass m and 3 m attached to its ends. The masses of string and pulley are negligible. When the system is released, its centre of mass moves with what acceleration ?

(a) 0 (b) – g/4 (c) g/2 (d) – g/2

25. A ring of mass m and radius *R* has four particles each of mass m attached to the ring as shown in figure. The centre of ring has a speed v_0. The kinetic energy of the system is



(a) mv_0^2 (b) $3mv_0^2$ (c) $5mv_0^2$ (d) $6mv_0^2$

26. Consider a uniform square plate of side '*a*' and mass '*M*'. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is

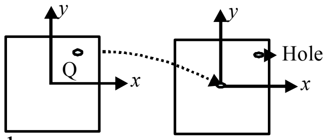
(a) $\frac{5}{6}Ma^2$ (b) $\frac{1}{12}Ma^2$ (c) $\frac{7}{12}Ma^2$ (d) $\frac{2}{3}Ma^2$

27. A dancer is standing on a stool rotating about the vertical axis passing through its centre. She pulls her arms towards the body reducing her moment of inertia by a factor of n. The new angular speed of turn table is proportional to

(a) n (b) n^{-1} (c) n^0 (d) n^2

28. A uniform square plate has a small piece Q of an irregular shape removed and glued to the centre of the plate leaving a hole behind. Then the moment of inertia about the z-axis

(a) increases
(b) decreases
(c) remains same
(d) changed in unpredicted manner.



29. A circular turn table has a block of ice placed at its centre. The system rotates with an angular speed ω about an axis passing through the centre of the table. If the ice melts on its own without any evaporation, the speed of rotation of the system

(a) becomes zero
(b) remains constant at the same value ω
(c) increases to a value greater than ω
(d) decreases to a value less than ω

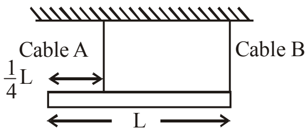
30. Seven identical coins are rigidly arranged on a flat table in the pattern shown below so that each coin touches it neighbors. Each coin is a thin disc of mass *m* and radius *r*. The moment of inertia of the system of seven coins about an axis that passes through point *P* and perpendicular to the plane of the coin is :

(a) $\frac{55}{2}mr^2$ (b) $\frac{127}{2}mr^2$ (c) $\frac{111}{2}mr^2$ (d) $55\,mr^2$

31. In a two-particle system with particle masses m_1 and m_2, the first particle is pushed towards the centre of mass through a distance d, the distance through which second particle must be moved to keep the centre of mass at the same position is

(a) $\frac{m_2 d}{m_1}$ (b) d (c) $\frac{m_1 d}{(m_1+m_2)}$ (d) $\frac{m_1 d}{m_2}$

32. A uniform bar of mass M and length L is horizontally suspended from the ceiling by two vertical light cables as shown. Cable A is connected 1/4th distance from the left end of the bar. Cable B is attached at the far right end of the bar. What is the tension in cable A?



(a) 1/4 Mg (b) 1/3 Mg (c) 2/3 Mg (d) 3/4 Mg

33. A couple produces

(a) purely linear motion
(b) purely rotational motion
(c) linear and rotational motion
(d) no motion

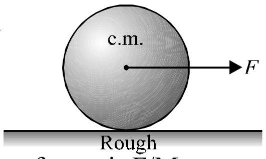
34. Point masses 1, 2, 3 and 4 kg are lying at the point (0, 0, 0), (2, 0, 0), (0, 3, 0) and (–2, –2, 0) respectively. The moment of inertia of this system about x-axis will be

(a) 43 kgm² (b) 34 kgm² (c) 27 kgm² (d) 72 kgm²

RESPONSE GRID					
	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ
	25. ⓐⓑⓒⓓ	26. ⓐⓑⓒⓓ	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ
	30. ⓐⓑⓒⓓ	31. ⓐⓑⓒⓓ	32. ⓐⓑⓒⓓ	33. ⓐⓑⓒⓓ	34. ⓐⓑⓒⓓ

Space for Rough Work

35. A solid sphere of mass M and radius R is pulled horizontally on a sufficiently rough surface as shown in the figure.

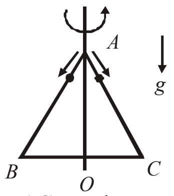


Choose the correct alternative.
(a) The acceleration of the centre of mass is F/M
(b) The acceleration of the centre of mass is $\frac{2}{3}\frac{F}{M}$
(c) The friction force on the sphere acts forward
(d) The magnitude of the friction force is F/3

36. The moment of inertia of a body about a given axis is 1.2 kg m^2. Initially, the body is at rest. In order to produce a rotational kinetic energy of 1500 joule, an angular acceleration of 25 radian/sec^2 must be applied about that axis for a duration of
(a) 4 sec (b) 2 sec (c) 8 sec (d) 10 sec

37. A gymnast takes turns with her arms and legs stretched. When she pulls her arms and legs in
(a) the angular velocity decreases
(b) the moment of inertia decreases
(c) the angular velocity stays constant
(d) the angular momentum increases

38. An equilateral triangle *ABC* formed from a uniform wire has two small identical beads initially located at *A*. The triangle is set rotating about the vertical axis *AO*. Then the beads are released from rest simultaneously and allowed to slide down, one along *AB* and the other along *AC* as shown. Neglecting frictional effects, the quantities that are conserved as the beads slide down, are

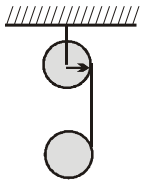


(a) angular velocity and total energy (kinetic and potential)
(b) total angular momentum and total energy
(c) angular velocity and moment of inertia about the axis of rotation
(d) total angular momentum and moment of inertia about the axis of rotation

39. The moment of inertia of a uniform semicircular wire of mass m and radius r, about an axis passing through its centre of mass and perpendicular to its plane is $mr^2\left(1-\frac{k}{\pi^2}\right)$. Find the value of k.
(a) 2 (b) 3 (c) 4 (d) 5

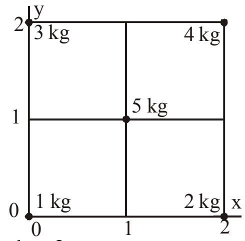
40. Initial angular velocity of a circular disc of mass *M* is ω_1. Then two small spheres of mass *m* are attached gently to diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc?
(a) $\left(\frac{M+m}{M}\right)\omega_1$ (b) $\left(\frac{M+m}{m}\right)\omega_1$
(c) $\left(\frac{M}{M+4m}\right)\omega_1$ (d) $\left(\frac{M}{M+2m}\right)\omega_1$.

41. Two identical discs of mass m and radius r are arranged as shown in the figure. If α is the angular acceleration of the lower disc and a_{cm} is acceleration of centre of mass of the lower disc, then relation between a_{cm}, α and r is



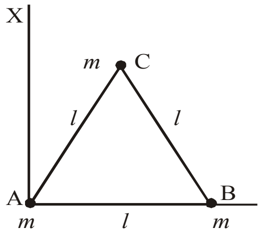
(a) $a_{cm} = \alpha/r$ (b) $a_{cm} = 2\alpha r$
(c) $a_{cm} = \alpha r$ (d) None of these

42. Five masses are placed in a plane as shown in figure. The coordinates of the centre of mass are nearest to



(a) 1.2, 1.4
(b) 1.3, 1.1
(c) 1.1, 1.3
(d) 1.0, 1.0

43. Three particles, each of mass *m* gram, are situated at the vertices of an equilateral triangle ABC of side ℓ cm (as shown in the figure). The moment of inertia of the system about a line AX perpendicular to AB and in the plane of ABC, in gram-cm^2 units will be



(a) $\frac{3}{2}m\ell^2$
(b) $\frac{3}{4}m\ell^2$
(c) $2\,m\ell^2$
(d) $\frac{5}{4}m\ell^2$

44. When a ceiling fan is switched on, it makes 10 rotations in the first 3 seconds. Assuming a uniform angular acceleration, how many rotation it will make in the next 3 seconds?
(a) 10 (b) 20 (c) 30 (d) 40

45. A solid sphere spinning about a horizontal axis with an angular velocity ω is placed on a horizontal surface. Subsequently it rolls without slipping with an angular velocity of :
(a) $\frac{2\omega}{5}$ (b) $\frac{7\omega}{5}$ (c) $\frac{2\omega}{7}$ (d) ω

RESPONSE GRID					
	35. ⓐⓑⓒⓓ	36. ⓐⓑⓒⓓ	37. ⓐⓑⓒⓓ	38. ⓐⓑⓒⓓ	39. ⓐⓑⓒⓓ
	40. ⓐⓑⓒⓓ	41. ⓐⓑⓒⓓ	42. ⓐⓑⓒⓓ	43. ⓐⓑⓒⓓ	44. ⓐⓑⓒⓓ
	45. ⓐⓑⓒⓓ				

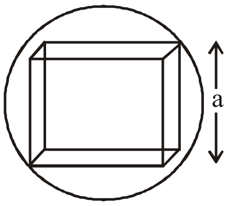
DAILY PRACTICE PROBLEM DPP CHAPTERWISE CP06 - PHYSICS

Total Questions	45	Total Marks	180
Attempted		Correct	
Incorrect		Net Score	
Cut-off Score	45	Qualifying Score	60
Success Gap = Net Score – Qualifying Score			
Net Score = (Correct × 4) – (Incorrect × 1)			

Space for Rough Work

DAILY PRACTICE PROBLEMS | PHYSICS SOLUTIONS | DPP/CP06

1. **(a)** Here $a = \frac{2}{\sqrt{3}}R$



Now, $\frac{M}{M'} = \frac{\frac{4}{3}\pi R^3}{a^3}$

$= \frac{\frac{4}{3}\pi R^3}{\left(\frac{2}{\sqrt{3}}R\right)^3} = \frac{\sqrt{3}}{2}\pi.\quad M' = \frac{2M}{\sqrt{3}\pi}$

Moment of inertia of the cube about the given axis,

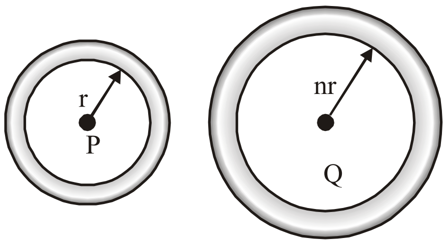
$$I = \frac{M'a^2}{6} = \frac{\frac{2M}{\sqrt{3}\pi}\times\left(\frac{2}{\sqrt{3}}R\right)^2}{6} = \frac{4MR^2}{9\sqrt{3}\pi}$$

2. **(d)** Initially centre of mass is at the centre. When sand is poured it will fall and again after a limit, centre of mass will rise.

3. **(a)** Does not shift as no external force acts. The centre of mass of the system continues its original path. It is only the internal forces which comes into play while breaking.

4. **(d)** Let the mass of loop P (radius = r) = m

So, the mass of loop Q (radius = nr) = nm

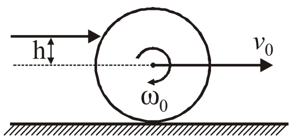


Moment of inertia of loop P, $I_P = mr^2$

Moment of inertia of loop Q, $I_Q = nm(nr)^2 = n^3mr^2$

$\therefore \frac{I_Q}{I_P} = n^3 = 8 \Rightarrow n = 2$

5. **(d)** When the ball is hit by a cue, the linear impulse imparted to the ball = change in momentum = mv_0

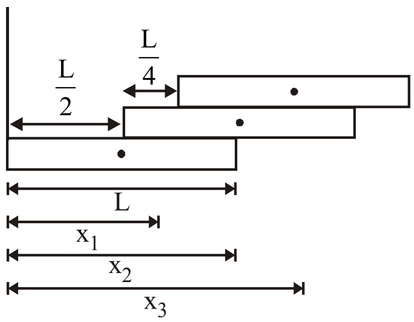


Angular momentum = Moment of momentum

$I\omega_0 = (mv_0)h$

$\frac{2}{5}mr^2\omega_0 = mv_0h$ or $\omega_0 = \frac{5v_0h}{2r^2}$

6. **(d)**



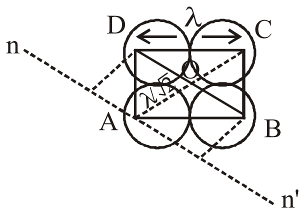
$x_1 = \frac{L}{2},\ x_2 = L,\ x_3 = \frac{5L}{4}$

$\therefore X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$

$= \frac{M\times\frac{L}{2} + M\times L + M\times\frac{5L}{4}}{M + M + M}$

$= \frac{\frac{11}{4}ML}{3M} = \frac{11L}{12}$

7. **(c)**

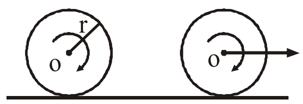


$I_{nn'}$ = M.I due to the point mass at B + M.I due to the point mass at D + M.I due to the point mass at C.

$I_{nn'} = 2\times m\left(\frac{\ell}{\sqrt{2}}\right)^2 + m(\sqrt{2}\ell)^2$

$= m\ell^2 + 2m\ell^2 = 3m\ell^2$

8. **(c)**



From conservation of angular momentum about any fix point on the surface,

$mr^2\omega_0 = 2mr^2\omega$

$\Rightarrow \omega = \omega_0/2 \Rightarrow v = \frac{\omega_0 r}{2}\quad [\because v = r\omega]$

9. **(d)** Initial position of cm $= \frac{m_2\ell}{m_1 + m_2}$

Also $x_{cm} = \dfrac{m_1\Delta x_1 + m_2\Delta x_2}{m_1+m_2} = \dfrac{m_1 v_0 t + 0}{m_1+m_2}$

$\therefore$ final position $= \dfrac{m_2\ell}{m_1+m_2} + \dfrac{m_1 v_0 t}{m_1+m_2}$

10. (a) Here, $L = 1.8\ kg\ m^2\ s^{-1}$, $M = 1.5\ kg$, $\omega = 0.3\ rad\ s^{-1}$

Angular momentum, $L = I\omega$

$L = k^2 M\omega$ $\quad (\because I = MK^2)$

or $1.8 = k^2 \times 1.5 \times (0.3)$

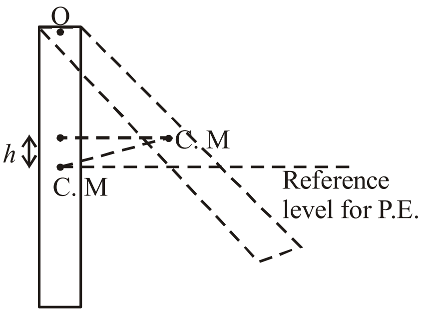
$\Rightarrow k^2 = \dfrac{1.8}{1.5 \times 0.3} = 4$

$\Rightarrow k = 2\ m.$

11. (b) $\vec{\tau} = \vec{r} \times \vec{F} \Rightarrow \vec{r}\,.\,\vec{\tau} = 0 \quad \vec{F}\,.\,\vec{\tau} = 0$

Since, $\vec{\tau}$ is perpendicular to the plane of $\vec{r}$ and $\vec{F}$, hence the dot product of $\vec{\tau}$ with $\vec{r}$ and $\vec{F}$ is zero.

12. (c)



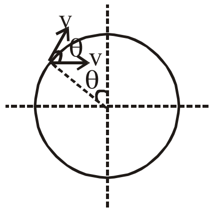
The moment of inertia of the rod about O is $\frac{1}{3}m\ell^2$.

The maximum angular speed of the rod is when the rod is instantaneously vertical. The energy of the rod in this condition is $\frac{1}{2}I\omega^2$ where I is the moment of inertia of the rod about O. When the rod is in its extreme portion, its angular velocity is zero momentarily. In this case, the energy of the rod is mgh where h is the maximum height to which the centre of mass (C.M) rises

$\therefore\ mgh = \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}\left(\dfrac{1}{3}ml^2\right)\omega^2$

$\Rightarrow\ h = \dfrac{\ell^2\omega^2}{6g}$

13. (b)



$v_R = \sqrt{v^2 + v^2 + 2v^2\cos\theta} = \sqrt{2v^2(1+\cos\theta)}$

$= 2v\cos\dfrac{\theta}{2}$

14. (b) $K.E_{rotational} = \dfrac{1}{2}I\omega^2$

$= \dfrac{1}{2}\dfrac{2}{5}\omega r^2 d^2 \quad \left(\because I_{Solid\ sphere} = \dfrac{2}{5}mr^2\right)$

$K.E_{translational} = \dfrac{1}{2}mv^2$

$\therefore\ \dfrac{K.E_{rotational}}{K.E_{translational}} = \dfrac{2}{5}$

Hence option (b) is correct

15. (c) Kinetic energy $_{(rotational)}\ K_R = \dfrac{1}{2}I\omega^2$

Kinetic energy $_{(translational)}\ K_T = \dfrac{1}{2}Mv^2$ $\quad (v = R\omega)$

M.I.$_{(initial)}\ I_{ring} = MR^2;\ \omega_{initial} = \omega$

M.I.$_{(new)}\ I'_{(system)} = MR^2 + 2mR^2$

$\omega'_{(system)} = \dfrac{M\omega}{M+2m}$

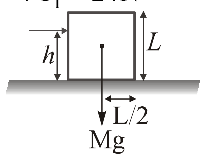
Solving we get loss in K.E.

$= \dfrac{Mm}{(M+2m)}\omega^2 R^2$

16. (b) For no angular acceleration $\tau_{net} = 0$

$\Rightarrow F_1 \times 5 = F_2 \times 30$ (given $F_2 = 4N$) $\Rightarrow F_1 = 24N$

17. (c) For toppling $Mg\dfrac{L}{2} = F_1 \times h$



For sliding

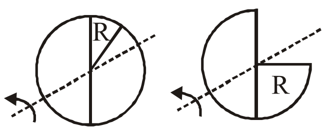
$\mu Mg = F_2$

For sliding to occur first

$F_1 > F_2$

or $\dfrac{mgL}{2} > \mu Mg$ or $L > 2\mu h$

18. (a)



(i) (ii)

Moment of inertia of a ring about a given axis is $I = MR^2$

Mass of the remaining portion of the ring $= \dfrac{3M}{4}$

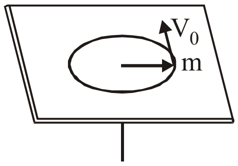
Moment of inertia of the remaining portion of the ring about a given axis is

$I' = \dfrac{3}{4}MR^2$

Given $I' = kMR^2$

$\therefore\ k = 3/4.$

19. (b) Applying angular momentum conservation

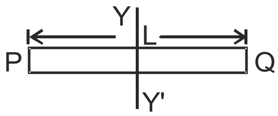


$$mV_0R_0 = (m)(V^1)\left(\frac{R_0}{2}\right)$$

$\therefore \quad v^1 = 2V_0$

Therefore, new KE $= \frac{1}{2}m(2V_0)^2 = 2mv_0^2$

20. (c) If rotation axis is passing through its middle point & is ⊥ to its plane, then moment of inertia about YY' is



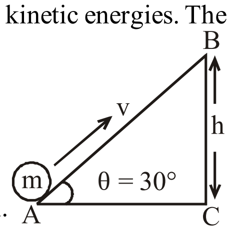
$I = \frac{ML^2}{12}$ where M = volume × density = (L×A)×ρ

so $I = \frac{L^3 A\rho}{12}$

so rotational K.E $= \frac{1}{2}I\omega^2 = \frac{L^3 A\rho\omega^2}{24}$

21. (c) If a body rolls on a horizontal surface, it possesses both translational and rotational kinetic energies. The net kinetic energy is given by

$$K_{net} = \frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right),$$



where K is the radius of gyration.
So from law of conservation of energy,

$$\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right) = mgh,$$

where h is the height attained by the sphere.

i.e., $\frac{1}{2}\times 2\times(10)^2\left(1+\frac{2}{5}\right) = 2\times 9.8\times h.$

i.e., $\frac{1}{2}\times 100\times\left(\frac{7}{5}\right) = 9.8h$

or $\quad h = \frac{700}{98} = 7.1m$

22. (c) After collision velocity of COM of *A* becomes zero and that of *B* becomes equal to initial velocity of COM of *A*. But angular velocity of *A* remains unchanged as the two spheres are smooth.

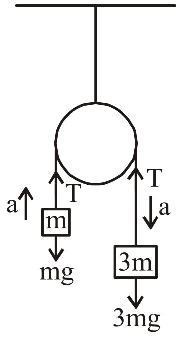
23. (b) M.I. of disc $= \frac{1}{2}MR^2 = \frac{1}{2}M\left(\frac{M}{\pi t\rho}\right) = \frac{1}{2}\frac{M^2}{\pi t\rho}$

$$\left(As\ \rho = \frac{M}{\pi R^2 t}\ \text{Therefore}\ R^2 = \frac{M}{\pi t\rho}\right)$$

If mass and thickness are same then, $I \propto \frac{1}{\rho}$

$$\therefore \frac{I_1}{I_2} = \frac{\rho_2}{\rho_1} = \frac{3}{1}$$

24. (c)



When the system is released, heavier mass move downward and the lighter one upward. Thus, centre of mass will move towards the heavier mass with acceleration

$$a = \left(\frac{3m-m}{3m+m}\right)g = \frac{g}{2}$$

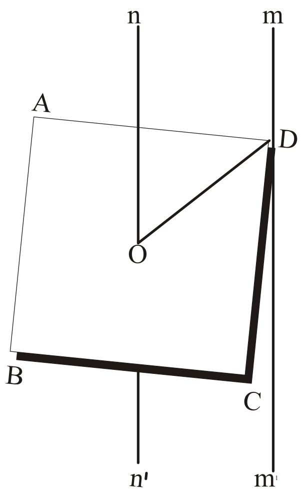
25. (c) $K = K_{ring} + K_{particles}$

$$= \left[\frac{1}{2}mv_0^2 + \frac{1}{2}I\omega^2\right] + \left[\frac{1}{2}m(\sqrt{2}v_0)^2 + \frac{1}{2}m(2v_0)^2 + \frac{1}{2}m(\sqrt{2}v_0)^2 + 0\right]$$

Also $\omega = \frac{v_0}{R}$ $\quad, I = mR^2$

$\therefore \quad K = 5mv_0^2$

26. (d) $I_{nn'} = \frac{1}{12}M(a^2+a^2) = \frac{Ma^2}{6}$



Also, $DO = \frac{DB}{2} = \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$

According to parallel axis theorem

$$I_{mm'} = I_{nn'} + M\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{Ma^2}{6} + \frac{Ma^2}{2}$$

$$= \frac{Ma^2 + 3Ma^2}{6} = \frac{2}{3}Ma^2$$

27. (a) From law of conservation of angular momentum,

$I\omega = I'\omega'$

Given $I' = I/n$

$\therefore \quad \omega' = n\omega \quad$ or $\quad \omega' \propto n$

28. (b)

29. (d) Melting of ice produces water which will spread over larger distance away from the axis of rotation. This increases the moment of inertia so angular velocity decreases

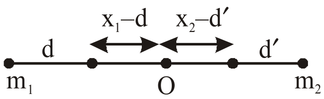
30. (c) $I_p = \frac{mr^2}{2} + 2\left[\frac{mr^2}{2} + m(2r)^2\right] + \left[\frac{mr^2}{2} + m(2r)^2\right]$

$+\left[\frac{mr^2}{2} + (2r)^2\right] + 2\left[\frac{mr^2}{2} + m(2\sqrt{3}r)^2\right]$

$= \frac{111}{2}mr^2$

31. (d) $0 = \frac{m_1(-x_1) + m_2x_2}{m_1 + m_2}$

$\therefore \quad m_1x_1 = m_2x_2$



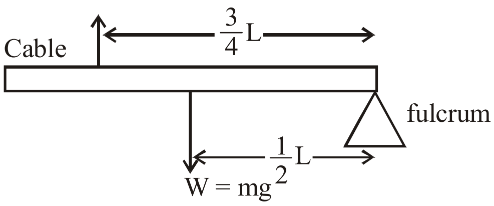
Now, $0 = \frac{-m_1(x_1 - d) + m_2(x_2 - d)}{m_1 + m_2}$

$0 = m_1(d - x_1) + m_2(x_2 - d')$

$\Rightarrow 0 = m_1d - m_1x_1 + m_2x_2 - m_2d'$

$\therefore \; d' = \frac{m_1}{m_2}d$

32. (c) This is a torque problem. While the fulcrum can be placed anywhere, placing it at the far right end of the bar eliminated cable B from the calculation. There are now only two forces acting on the bar ; the weight that produces a counterclockwise rotation and the tension in cable A that produces a clockwise rotation. Since the bar is in equilibrium, these two torques must sum to zero.



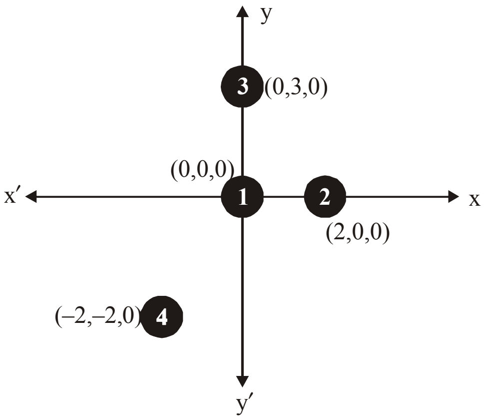
$\Sigma\tau = T_A(3/4L) - Mg(1/2L) = 0$

Therefore

$T_A = (MgL/2)/(3L/4) = (MgL/2)(4/3L) = 2Mg/3$

33. (b) Couple produces purely rotational motion.

34. (a)



$I_1 = I_2 = 0$, because these particles are placed on x-axis

The M.I. of system about x-axis, $= I_1 + I_2 + I_3 + I_4$

$= 0 + 0 + 3 \times (3)^2 + 4 \times (-2)^2 = 27 + 16 = 43 \text{ kg} - m^2$

35. (b)

36. (b) $I = 1.2 \text{ kg m}^2, E_r = 1500 \text{ J}$,

$\alpha = 25 \text{ rad/sec}^2, \omega_1 = 0, t = ?$

As $E_r = \frac{1}{2}I\omega^2, \omega = \sqrt{\frac{2E_r}{I}} = \sqrt{\frac{2 \times 1500}{1.2}} = 50 \text{ rad/sec}$

From $\omega_2 = \omega_1 + \alpha t$

$50 = 0 + 25t, \quad \therefore \quad t = 2$ seconds

37. (b) Since no external torque act on gymnast, so angular momentum ($L = I\omega$) is conserved. After pulling her arms & legs, the angular velocity increases but moment of inertia of gymnast, decreases in such a way that angular momentum remains constant.

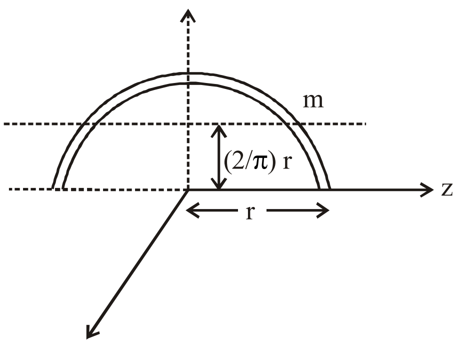
38. (b) The *M.I.* about the axis of rotation is not constant as the perpendicular distance of the bead with the axis of rotation increases.

Also since no external torque is acting.

$\therefore \tau_{ext} = \frac{dL}{dt} \Rightarrow L = \text{constant} \Rightarrow I\omega = \text{constant}$

Since, I increases, ω decreases.

39. (c)



Moment of inertia about z-axis, $I_z = mr^2$

(about centre of mass)

Applying parallel axes theorem,

$I_z = I_{cm} + mk^2$

$I_{cm} = I_z - m\left(\frac{2}{\pi}r\right)^2 = mr^2 - \frac{m4r^2}{\pi^2} = mr^2\left(1 - \frac{4}{\pi^2}\right)$

i.e., $k = 4$

40. **(c)** When two small spheres of mass m are attached gently, the external torque, about the axis of rotation, is zero and therefore the angular momentum about the axis of rotation is constant.

$\therefore\ I_1\omega_1 = I_2\omega_2 \Rightarrow \omega_2 = \frac{I_1}{I_2}\omega_1$

Here $I_1 = \frac{1}{2}MR^2$

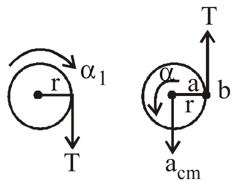
and $I_2 = \frac{1}{2}MR^2 + 2mR^2$

$$\therefore\ \omega_2 = \frac{\frac{1}{2}MR^2}{\frac{1}{2}MR^2 + 2mR^2}\times\omega_1 = \frac{M}{M+4m}\omega_1$$

41. **(b)** $Tr = \frac{mr^2}{2}\alpha_1$ .... (1)

$Tr = \frac{mr^2}{2}\alpha$ .... (2)

$\alpha_1 = \alpha$ .... (3)



Acceleration of point b = acceleration of point a

$r\alpha_1 = a_{cm} - r\alpha$ .... (4)

Hence, $2r\alpha = a_{cm}$

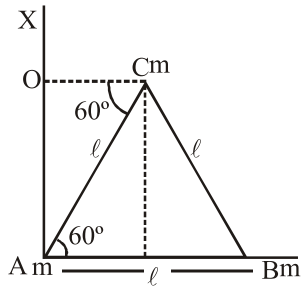
42. **(c)** $X_{C.M.} = \frac{1\times0+2\times2+3\times0+4\times2+5\times1}{1+2+3+4+5}$

$= \frac{4+8+5}{15} = \frac{17}{15} = 1.1$

$Y_{C.M} = \frac{1\times0+2\times0+3\times2+4\times2+5\times1}{1+2+3+4+5}$

$= \frac{6+8+5}{15} = 1.3$

43. **(d)** $I_{AX} = m(AB)^2 + m(OC)^2 = m\ell^2 + m(\ell\cos 60^\circ)^2$
$= m\ell^2 + m\ell^2/4 = 5/4\, m\ell^2$



44. **(c)** Angle turned in three seconds, $\theta_{3s} = 2\pi\times10 = 20\pi$ rad.

From $\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \Rightarrow 20\pi = 0 + \frac{1}{2}\alpha\times(3)^2$

$\Rightarrow \alpha = \frac{40\pi}{9}$ rad/s^2

Now angle turned in 6 sec from the starting

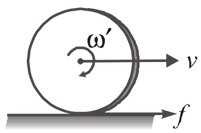
$\theta_{6s} = \omega_0 t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}\times\left(\frac{40\pi}{9}\right)\times(6)^2 == 80\pi$ rad

$\therefore$ Angle turned between $t = 3$s to $t = 6$s

$\theta_{last\ 3s} = \theta_{6s} - \theta_{3s} = 80\pi - 20\pi = 60\pi$

Number of revolutions $= \frac{60\pi}{2\pi} = 30$.

45. **(c)** $a = \frac{f}{m} = \mu g$



and $\alpha = \frac{fR}{I} = \frac{\mu mgR}{\frac{2}{5}mR^2} = \frac{5}{2}\frac{\mu g}{R}$

Now $v = 0 + at$

and $\omega' = \omega - \alpha t$

Also $\omega' = \frac{v}{R}$

After solving above equations, we get $\omega' = \frac{2\omega}{7}$